The World Description Made in *Zhou bi Suanjing* Has Been Revealed

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ABSTRACT: A millenary description of the world as conceived by Chinese culture, appears in the *Zhou bi Suanjing* or "The book of the Gnomon and the Circular Paths of the Heaven", according Needham. Situated at Zhou (a legendary city) the dimensions of the earth and heaven, including the size of the Sun are presented in this book. The Chinese Gaì Ti'an astronomical model of a flat earth and a flat heaven, is measured with the aid of a gnomon (*biao*) and the application of the Pythagorean Theorem. Now, after making an adjustment to the scale of the data, the elements to locate the legendary Zhou City have been found making feasible the correspondence between the dimensions of the Zhoubi's given world and those of the actual Earth. Here I show that Zhou, centre of the world from which the measures are done, corresponds to the city Jining located in the province of Shandong, China; also, I confirm that the distance between Jining and the Pole of *Zhoubi* is $4,285\ km$ as could be found with a geographical tool such as the Google-Earth program. With these results I am confident to say that: The city of Zhou appearing in the *Zhoubi* exists; and, that the *Zhoubi* was an accurate description of the Earth at the time of Zhou Dynasty (1045 - 256 BC). Moreover, the location of cities as important as Beijing (at 498 km to the north of Jining) would be related with projection of the heaven on

Earth: it is about one diameter of Sun from Jining. Also, it important to know that this city is recognized as the birth place of the great philosopher Confucius, giving to my findings a relevance that goes beyond the field of Astronomy or Mathematics, in which frame the Zhoubi has been analysed through the years. These findings put new perspectives for the search in other non-mathematical or non-astronomical documents elements for these disciplines.

KEY-WORDS: instrumentation: miscellaneous — methods: observational — Sun: general — Earth — celestial mechanics — history and philosophy of astronomy — Chinese culture

1. INTRODUCTION

The *Zhoubi* (short name of the book) is a compilation of texts describing the astronomical knowledge at the epoch of the Zhou Dynasty and possibly earlier (Needham 1958). It contains an interrelated world represented by a flat earth where the people live and a flat heaven where the Pole and the Circular Paths of the Sun around it are located: The Chinese Gaì Ti'an astronomical model (Chen 1996). This depiction of the world is based in three fundamental facts that can be considered as being observed: the shadows of a gnomon measured at *Zhou*; when pointing to the Pole with a cord and a gnomon, its position could be measured; and, the size of the Sun can be estimated using a bamboo tube (Cullen 2007). These observations are combined with other assumptions making the global description of the world feasible; these assumptions could not be considered necessarily as observable facts: The length of the shadow changes at a certain rate in relation to the change in the position of the Sun; the separation between Heaven where the Sun, Moon and stars move, and the Earth below is constant and can be measured to be 80.000 *li*; and, the place just below the Pole (north) is located at 103.000 *li* distance from *Zhou* city.

The actual lengths represented by the *li* and *chi* units has been changing in time, but we can consider its values to be around 0,358 - 0,416 km and 0,199 - 0,202 m, respectively (Martzloff 1997). If one considers these values, the dimensions of the

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world described in the *Zhoubi* would be those presented in the first columns of table 1. It is evident that the mentioned earth values are no feasible for the Earth we live on. More important, they are not compatible with the text in the sense that, for example, while talking about noon shadows, it says

– #B9 [26a] '16 000 li to the south at the summer solstice, and 135 000 li to the south at the winter solstice, if one sets up a post (*gan*) at noon it casts no shadow. This single [fact is the basis of] the numbers of the Way of Heaven.' (Cullen 2007)

Which imply that it would be possible to reach that point. However, one must walk 6.656 km to arrive at the place where at the summer solstice there are no shadows at noon. The Pole would be located almost 45.000 km to the north from *Zhou*. Both distances are far from being possible.

In this paper, I consider an alternative interpretation of the data. I suggest that the values of the shadows (13,5 and 1,6 *chi* for the WSs and SSs, respectively) could have been observed in some past time as determined by Yong Li and Xiao-Chun Sun, in the exact North-South direction (Li & Sun 2009) but considering actual measures that do not need any correction; and, that they were used in later times to define the dimensions of the ancient known world. I consider the proposal of a correction in the above mentioned assumptions made based on the comparison between dimensions of the Earth and those given in the *Zhoubi* book.

2. THE WORLD IN ANCIENT CHINA

The *Zhoubi* is a compilation of texts describing the astronomical knowledge at the epoch of the Zhou Dynasty and few centuries before (Needham 1958). The parallel flat surfaces for Earth and Heaven is the depiction of the world that I analyse in terms of the description given in the *Zhoubi*; as mentioned above, it is based in three fundamental observable facts in which my hypothesis will take into account; all of them, explicitly given in the *Zhoubi* when a gnomon of 8 *chi* is used (Cullen 2007):

– a) the shadows of a gnomon located at a place named *Zhou* were 13,5 and 1,6 *chi* in length at winter solstice (WSs) and at summer solstice (SSs), respectively;
– b) when pointing to the Pole (region of the sky around which Sun and stars revolve), with the help of a cord and a gnomon, the 'shadow' is 10,3 *chi*; and,
– c) The size of the Sun can be fixed using a bamboo tube of 8 *chi* in length and 0,1 *chi* of internal diameter, when the Sun casts a shadow of 6 *chi* at noon.

These observations are combined, in the text of the book, with other assumptions making the global description of the world feasible but not very accurate:

d) The length of the shadow cast by a gnomon changes at a rate of 0,1 *chi* for each 1 000 *li* change in the position of the Sun;
e) The Heaven where the Sun lies is 80 000 *li* above the Earth; and,
f) The point just below the Pole (north) is located at 103 000 *li* distance from *Zhou*.

Then, for example, one must walk 6,656 *km* to arrive at the place where, at the summer solstice, there are no shadows at noon. The Pole would be located almost 45,000 *km* to the north of *Zhou*. Could these distances represent actual dimensions of the Earth? See Table 1.

Table 1 Comparing Dimensions of the World

Position[1]	Distance[2] (*li*)	Distance[2] (km)	NewDistance[3] (*li*)	NewDistance[3] (km)
Pole	103 000	42 848 - 36 874	10 300	4 285 - 3 687
Winter Solstice	135 000	56 160 - 48 330	13 500	5 616 - 4 833
Summer Solstice	16 000	6 656 - 5 728	1 600	666 - 573
Sun's date	60 000	24 960 - 21 480	6 000	2 496 - 2 148
Sun's diameter[4]	1 250	520 - 447.5	1 250	520 - 448

[1] Point on Earth just below heaven.
[2] Distance from Zhou City as reported in Zhoubi.
[3] Distance from Zhou city as corrected by using a 100 *li* by *cun*
[4] The Sun's diameter is the same for both cases.

However, more feasible values are found if one assumes that the statement d) above presented is adjusted to say that 0,1 *chi* in shadow imply a 100 *li* in Sun's distance (one *chi* instead of one *cun*) (Cullen 1976; 124). With this in mind I calculated the more reasonable values given in columns 4th and 5th of Table 1. It is important to note that the proposed change does not modify a measurable fact but makes one of the assumptions given more suited to the size of the Earth.

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In the forthcoming part of this paper the reason of this change will be self-evident. The height of heaven has been adjusted in concordance: 8.000 *li* instead of 80.000 *li*. These changes are needed in order to make feasible the use of the Pythagorean Triangle as it is done in the book I am analysing.

Once the reported distances have been made compatible with those of the Earth, for example, the Pole now can be actually reached, I can follow to look forward to find the site where the legendary site of *Zhou* was located:

> "Rong Fang asked: 'What is Zhou-Bi? Chàn Zi replied: 'It was that used by ancient high king to relate [the seasons] in *Zhou*. The mathematical art originated from the *Zhou*, and is, therefore, called *Zhou-Bi*. Bi means gnomon.'"(Cullen 2007)

Again, it will be necessary to use the observable values for the shadows cast by the gnomon: knowing that a gnomon of 8 *chi* in height (*g*) was used. The shadows cast by a gnomon depend on its height and on the elevation of the Sun above the horizon. We know that solstices correspond to the maximum declination of the Sun at noon due to the obliquity of the ecliptic. They depend also on the latitude of the place where the observation is made. Then, I have approach myself to the actual position and epoch of observation through the following relations, in a similar way as I did few years ago for the Oracle of Dodona (Perez-Enriquez 2014) and as discussed by Yong Li and Xiao-Chun Sun (Li & Sun 2009).

Let α and β be the Sun's elevation at solstices (WS and SS, respectively), then

$$WSs = \frac{g}{\tan(\alpha)}, \qquad SSs = \frac{g}{\tan(\beta)}, \qquad (1)$$

with

$$\alpha = 90° - \phi - \theta, \qquad \beta = 90° - \phi + \theta, \qquad (2)$$

Where the latitude of observation is ϕ and the obliquity of ecliptic is θ. It is strait forward to estimate both values from the following expressions:

$$\phi = \tfrac{1}{2}\left[180° - \left(\tan^{-1}\left(\frac{g}{WSs}\right) + \tan^{-1}\left(\frac{g}{SSs}\right)\right)\right], \qquad (3)$$

and

$$\theta = \tfrac{1}{2}\left[\left(\tan^{-1}\left(\frac{g}{SSs}\right) - \tan^{-1}\left(\frac{g}{WSs}\right)\right)\right]. \qquad (4)$$

With the Latitude found, the search for the possible location of *Zhou,* the place where measurements were made, can be done easily with the aid of a tool like Google Earth (GoogleEarth 2017). We just need to look at a fringe around $\phi = 35.33^o$ in the region of China. It was very surprising for me to find out that one of such possible sites is precisely the city of Jining in Shandong. More astonishing was to realize that this was the birth city of Confucius, the great philosopher (Wikipedia 2017).

In Figure 1 a map of the region centred at Jining is presented. As it is possible to see, the place and its surroundings are in harmony with the description given in the *Zhoubi*. I have identified eight cities almost equidistant to Jining that are located pointing to the directions to the rising and setting of the Sun at the calendrical dates: Winter Solstice, Summer Solstice and Equinoxes and at North and South (see section #J of *Zhoubi*) (Cullen 2007). In Table 2, I show the distances on the map to these cities.

If one follows north beyond Botou, one arrives to Beijing (Peking in the figure). Again I must say that I got a value in correspondence with those of Table 1; it is located at about 1 250 *li* from *Zhou*: The Imperial Capital and centre of China is found at a distance of around one diameter of the Sun! ($498.1\ km$ in Table 2). According to *Zhoubi*. The Sun's observed diameter at the day when the gnomonic triangle is the Pythagorean Triangle 6:8:10 (Cullen 2007).

The Pole, the place on Earth just below that in the heaven around which the circular paths of the Sun contract and expand through the year, was located at 10,300 *li* form *Zhou* (4,285 km to the north), matching the farthest point on earth that can be reached in the Eurasian Continent as it is shown in Figure 2; it is a place located near the coast between the Gulf of Jalanga and the Gulf of Oleniok, about 879 km from true North pole (GoogleEarth 2017).

With these results I am confident to say that the legendary *Zhou* reported in the *Zhou bi Suanjing* could be the city of Jining and it may be considered as part of a fairly accurate description of the world known at the time of Zhou Dynasty; the *Zhoubi* may represent a map of the Earth and heaven with the latter projected on the former: "That which is below is like that which is above and that which is above

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is like that which is below...", used to say Isaac Newton (Dobbs 1988). Finally, the meaning of *Zhou* as the city where the great philosopher Confucius was born (City of Jining) need a different approach which I leave to a more philosophical paper. Further analysis will be presented in the near future.
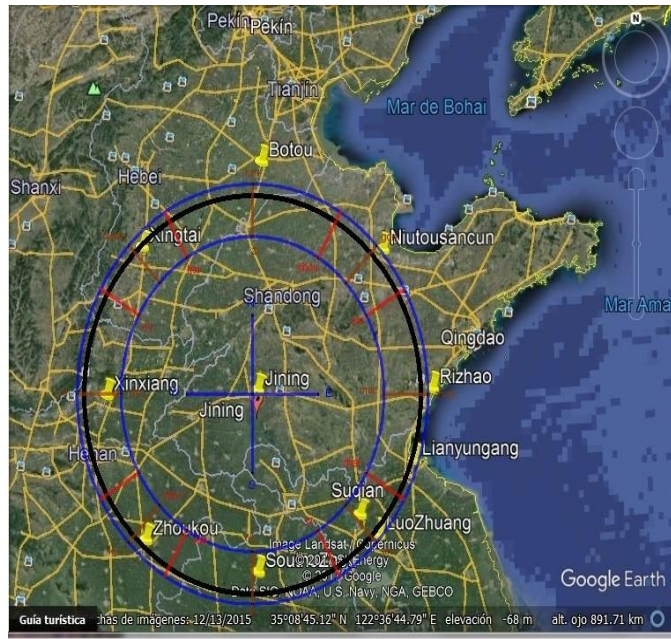

Fig.1 View of the region centred at Jining, legendary *Zhou* city.
About 500 km to the north appears Beijing (GoogleEarth).

Table 2 Cities and places around Jining compatible with *Zhoubi*

Site[1]	Az (?)	Pointing[6]	Epoch[2] D[3] (km)	Eastern Zhou D (*li*)	Western Zhou D (*li*)	Zhoubi prop D (*li*)
			(km/li)	0.416	0.358	0.399
Lingxian	0.02	kan	215.5	518	602	540
Botou	359.80	kan	296.2	712	827	742
Laiwu	45.00	gen	136.7	329	382	342
Niutousancun	45.30	gen	263.7	634	737	660
Rizhao	89.24	zhen	264.9	637	740	663
LuoZhuang	135.20	sun	262.7	631	734	658
Suqian	135.80	sun	222.8	536	622	558
Mengcheng	180.40	li	239.6	576	669	600
Zhoukou	222.20	kun	261.2	628	730	654
Xinxiang	267.80	dui	241.8	581	597	605
West of Zhou	270.00	dui	264.2	635	652	662
Xingtai	315.60	quian	262.3	631	733	657

Beijing[4]	357.94	498.1	1197	1391	1248
Pole[5]	0.05	4285.0	10300	11969	10732
North pole	0.05	6164.0	14817	15220	15438

1 Places on the map of figure1.

2 Three equivalences between li and km.

3 Distances from Jining in kilometres (km).

4 Capital at the same distance as the diameter of the Sun.

5 Same as the distance from Jining to the middle point between Guls of Jalanga and Oleniok.

6 Names of the zones in the twelve-fold azimuthal division of the horizon in #J of Zhoubi (Cullen).
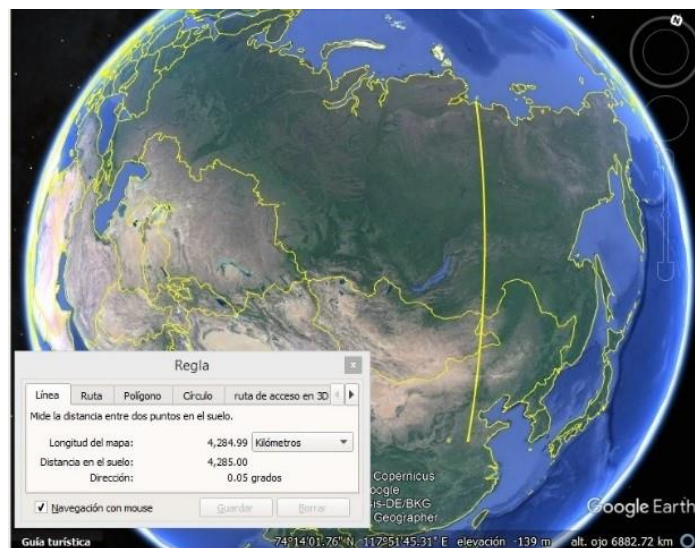

Fig.2 Map of the Earth with a measure of the distance between Jining and the Pole (GoogleEarth).

R.P.-E.

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